

Linearteam technical paper

Electroacoustic modelling of the subwoofer enclosures

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July 6, 2007

1 Introduction

Motivation for writing this document is that there are very little information about equivalent circuit modelling freely available, so this is my contribution to this field and I hope information represented in this document will prove useful, at least some way. Finnish HIFI-magazine (now gone to oblivion) seems to be unwilling to release any detailed information (I have personally read some questions and blunt answers to them), I suspect that they don't want to "waste" valuable pages to explaining rather electro-mathematical subject. Instead they have been suggesting that you should buy a design software to calculate these things. But that answer does not help much if you want to develop insight knowledge what is going on "under the hood". This document also serves as a quick reference for me. And, because it is available on the web, I can easily dig this out if I need to remember something.

I have developed this knowledge during developing new enclosure models for Juha Hartikainen's excellent freeware speaker design program, WinISD Pro. I gathered this document from my handwritten notes. It actually turned out that it is possible to calculate so many things from these models, so representing them all rationally is difficult. But in time, I hope that more graphs will appear on WinISD Pro. Please feel free to send any corrections, additions, feedback, suggestions about this document.

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2 Bits and pieces

2.1 Analogies to circuit theory

In these models, the current through circuit branch represents volume velocity. Voltage at particular node represents acoustic pressure. Note that these models work only when box is small compared to the wavelength. So you can't describe a transmission line properly by using these components. That is actually true on electrical circuits, but there usable frequency range is much higher (electrical signals travel much faster than acoustic waves).

2.2 Acoustic quantities

Here are the quantities, that electrical values represent in acoustic circuits

$$[U] = \frac{\text{N}}{\text{m}^2} = \text{Pa} \quad (1)$$

$$[I] = \frac{\text{m}^3}{\text{s}} \quad (2)$$

$$[R] = \frac{[U]}{[I]} = \frac{\text{Ns}}{\text{m}^2\text{m}^3} = \frac{\text{Ns}}{\text{m}^5} \quad (3)$$

Complex frequency is denoted with s , which is defined as usual

$$s = j\omega \quad (4)$$

Wavenumber, k is defined as usual,

$$k = \frac{\omega}{c} \quad (5)$$

2.3 Volume velocity

Air displaced by piston source is denoted as $V = S_D x$. Then, volume velocity emitted is defined as

$$U = \frac{dV}{dt} = S_D \frac{dx}{dt} = S_D u \quad (6)$$

Where U is volume velocity and u is velocity of piston.

2.4 Calculating air properties

If you want to split hairs with sound velocity use this formula to calculate sound velocity at desired temperature:

$$c = \sqrt{\frac{\gamma RT}{M}} \quad (7)$$

where T is absolute temperature in Kelvins, M is molar mass of gas in question (for air, M is about $29 \times 10^{-3} \text{kg/mol}$), R is molar gas constant, about $8.3145 \text{J}/(\text{K} \cdot \text{mol})$, γ is adiabatic

constant, ratio of specific heats, for air γ is about 1.40 and c is propagation velocity of sound¹. Additional difficulties arise from the fact that, air contains also water vapour. For more detailed discussion, see Claus Futtrup's DPC[1]. WinISD Pro uses the same advanced air model than DPC does. Throughout this document, values of c and ρ are 343.68 m/s and 1.20 kg/m³, unless stated otherwise.

2.5 Piston radiation impedance in infinite baffle

Radiation impedance of flat piston in infinite baffle is defined to be

$$Z_{\text{RAD}} = \frac{1}{S_D^2} \frac{f}{u} = \frac{\rho c}{\pi a^2} \left[\left(1 - \frac{J_1(2ka)}{ka} \right) + j \frac{H_1(2ka)}{ka} \right] \quad (8)$$

where the J_1 is the Bessel function of first kind of order one, and H_1 is the Struve function of first kind. a is the radius of the piston. This function is very difficult to evaluate without a computer or tabulated values for the Struve and Bessel functions. Real and imaginary parts are plotted in the figure 1. The Struve function is here approximated by the method described by Aarts and Augustus[2]

$$H_1(z) \approx \frac{2}{\pi} - J_0(z) + \left(\frac{16}{\pi} - 5 \right) \frac{\sin z}{z} + \left(12 - \frac{36}{\pi} \right) \frac{1 - \cos z}{z^2} \quad (9)$$

which is quite accurate, better than 0.5 % in most cases.

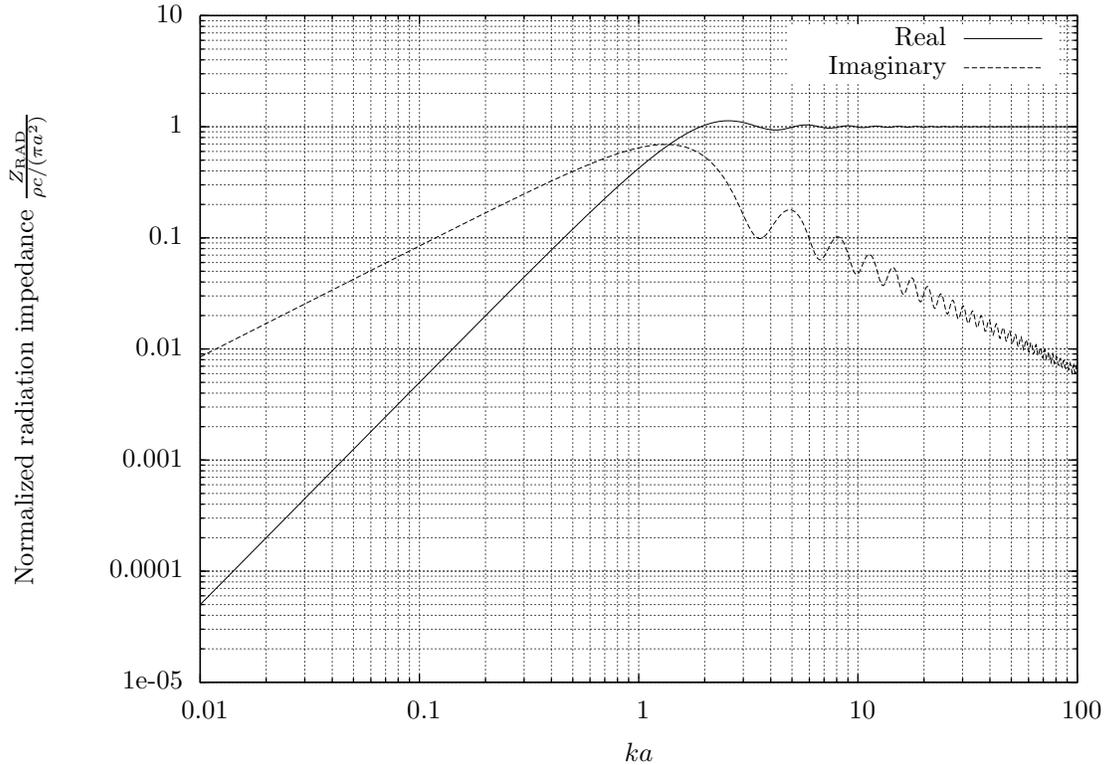


Figure 1: Piston radiation impedance in the infinite baffle.

¹It is assumed, that SI-units are used, so volume is in m³, mass in kg's, forces in Newtons, areas in m² etc.

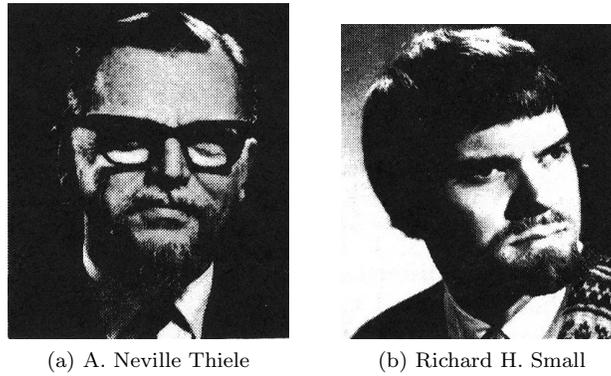


Figure 2: Developers of systematic design of ported boxes.

As can be seen from figure 1, the resistive (real) part is very small at low frequencies. This is main reason why normal direct radiator loudspeakers are notoriously inefficient. The reactive part of the radiation impedance describes the extra mass which the piston (driver) appears to have when it is vibrating. To have a perspective to normally used range, if we have a driver which has a S_D of about 550 cm^2 ($12''$) ($\Leftrightarrow a \approx 132 \times 10^{-3}$) and used frequency range is $20 < f < 100$ Hz, then the ka is in range of 0.048 - 1.828. Thus for for the most typical subwoofer frequency range, we have $ka < 1$. In practice, the radiation impedance is approximated in analogous circuits by simple circuits which have correct asymptotic behaviour with quite good accuracy. Radiation impedance is so small that if the actual efficiency η is not required to be calculated, then for the response calculations, it can be replaced by a short without making a gross error.

2.6 Thiele, Small, Who the heck?

You have probably heard people talking about Thiele and Small. They were among the first to derive modern synthesis methods to speaker design. Small[3] also represented complete and solid theoretical basis for direct-radiator loudspeaker analysis.

2.7 Series and parallel resonance circuit quality factors

Quality factor of series RLC resonance circuit is defined as

$$Q_{\text{ser}} = \frac{1}{\omega RC} \quad (10)$$

whereas parallel resonance RLC circuit is defined as

$$Q_{\text{par}} = \omega RC \quad (11)$$

These are useful results when calculating equivalent circuit component values.

3 Driver and passive radiator model

3.1 Driver basic models

Electrodynamic driver can be simulated using electrical or acoustical equivalent circuit. The driver motor is most conveniently modelled as an gyrator, which behaves like bidirectional transconductance. That means that roles of current and voltages are exchanged[4]. This may sound weird, but it is quite easy concept to grasp. The basic formula of the wire in the magnetic field says that force \vec{F} produced by current i in length \vec{l} in magnetic field \vec{B} is

$$\vec{F} = (\vec{B} \times \vec{l}) i \quad (12)$$

In practice, the magnetic field vector \vec{B} and length vector \vec{l} are assumed to be perfectly perpendicular to each other, and we can use scalar values for the B and l variables. Thus, the magnitude of vector cross product $\vec{B} \times \vec{l}$ is called the *force factor* in the electroacoustics literature, denoted by just Bl . The convention is to set polarities so that positive current into driver positive terminal produces positive velocity of the diaphragm, i.e. it moves out of the driver.

Since the force is appearing as voltage and velocity is like current, it can be seen from (12) that velocity(current) becomes force(voltage). This leads to so called *impedance analogy*. Conventional ideal transformer could be used instead of the gyrator, but then we would have to deal with not-so-nice mobility analog, where current represents force and voltage represents as velocity, which is somewhat counterintuitive, at least to me.

Electrical inductance creates a voltage which resists the *change* of current through the inductor. Similarly, mass resists changing of it's velocity by generating a force. Electrical capacitance tends to keep constant voltage and generating a current resisting the change of the voltage. Again, a mechanical spring resists changing force across it by generating a velocity.

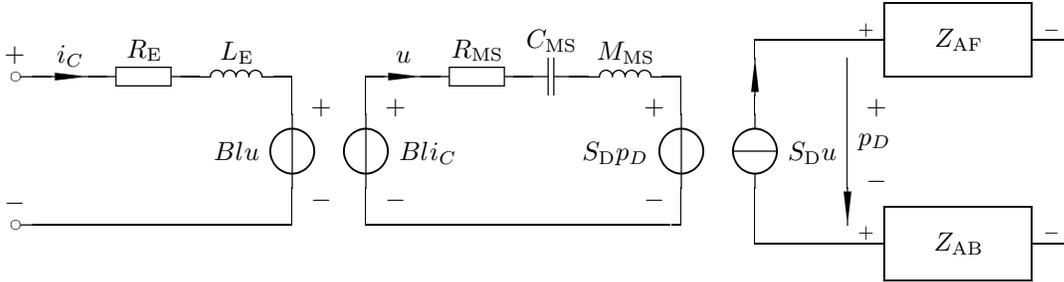


Figure 3: Electrodynamic driver seen from electrical side.

This is illustrated in generalized driver electrical equivalent circuit with typical load impedances in figure 3. Force Bli_C exerts force on the driver diaphragm, which then starts to move at velocity u . This creates back-EMF of Blu , which is seen as rise of electrical impedance at driver voice coil terminals. More freely the diaphragm moves, higher the impedance. When the diaphragm moves, it generates a volume velocity $S_D u$. Due to air load and possible loading from the box where the driver is mounted, this creates a counterforce $S_D p_D$, which in turn resists movement of the diaphragm. This is seen as reduction in the driver electrical impedance. Z_{AB} is the acoustic load impedance of the box. Z_{AF} is free-air load impedance, which contains the radiation resistance. The power dissipated to the radiation resistance is the part of electrical input power what actually gets converted to sound. The most part of the electrical input power is just dissipated in R_E to heat. Note that not all box types couple the driver to free air. Bandpass boxes typically contain the driver in between of two chambers, and the sound is radiated from one or more Helmholtz resonator port(s).

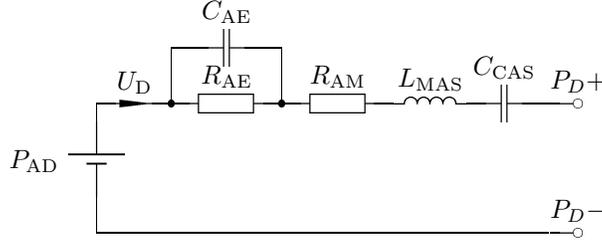


Figure 4: Driver equivalent circuit reduced to acoustical side.

For most simple simulations, which do not require access to electrical terminals, this model can be reduced to combination analog circuit to acoustical side, shown in figure 4. Thus circuit that contains just constant pressure generator P_{AD} , which depends upon e_G .

3.2 Determination of parameters for combination analog circuit

The driver is modelled as a series-RLC circuit with voltage source, where the resistance describes overall losses introduced by the driver. Inductor represents moving mass of the diaphragm (including air load). Capacitor represents compliance of driver suspension.

One possible procedure for calculating actual numerical parameters for these components would be, for example following way. Driver components can also be calculated by from electro-mechanical parameters. This approach is also shown. It is also possible to use direct electro-mechanical model using VCCS-component in SPICE.

1. Calculate value of C_{CAS} :

This capacitor depends solely upon driver V_{AS} and surrounding air properties, thus it is given by the following relation:

$$C_{CAS} = \frac{V_{AS}}{\rho c^2} = C_{MS} S_D^2 \quad (13)$$

where c is velocity of sound in air (about $343.68 \frac{m}{s}$), and ρ is density of air ($1.20 \frac{kg}{m^3}$), assuming the same conditions.

This dependancy on air properties is the fact that makes me nervous. It implies, that there is no absolutely correct value of V_{AS} existing, and never will be. Though, the C_{MS} is also quite unstable parameter, due to change of the physical properties of the suspension material.

2. Calculate L_{MAS}

This inductor represents moving mass of the driver, but if M_{MS} is not available, M_{MS} can be derived from f_s and equivalent volume V_{AS} , by noting that C_{CAS} and L_{MAS} form a series RLC-circuit, which has resonant frequency equal to driver f_s . By using the well-known formula for LC-resonance circuit, we obtain

$$f_s = \frac{1}{2\pi\sqrt{L_{MAS}C_{CAS}}} \Leftrightarrow L_{MAS} = \frac{1}{(2\pi f_s)^2 C_{CAS}} \quad (14)$$

Or, more directly

$$L_{MAS} = \frac{M_{MS}}{S_D^2} \quad (15)$$

Note that M_{MS} must be with air load included. Air load mass will change if driver is mounted different way. Usually, it won't be much off anyway.

If you want to simulate effect of extra mass M_e in driver, then you can increase L_{MAS} by amount L_{ME} :

$$L_{ME} = \frac{M_e}{S_D^2} \quad (16)$$

Then just add L_{ME} to L_{MAS} to obtain total acoustical inductance.

3. Calculate R_{AE} and R_{AM}

First, adjust Q_{ES} value to include any external resistance seen by the driver. This is done by multiplying the original Q_{ES} with factor, thus we obtain actual Q_{ES}' , which is denoted with Q_{ES}' :

$$Q_{ES}' = \frac{R_g + R_E}{R_E} Q_{ES} \quad (17)$$

where R_g is total sum of all resistances seen by driver. Therefore, it must include wires, connectors, amplifier output resistance, crossover inductor DC resistances etc.

Now, we can calculate R_{AE} , which represents electrical losses converted to acoustic side, so by combining the series RLC-circuit Q definition (10) with RLC-resonance circuit frequency formula we obtain

$$\begin{aligned} Q_{ES}' &= \frac{1}{\omega C_{CAS} R_{AE}} = \frac{\sqrt{L_{MAS} C_{CAS}}}{C_{CAS} R_{AE}^2} = \sqrt{\frac{L_{MAS}}{C_{CAS} R_{AE}^2}} \\ &\Leftrightarrow R_{AE} = \sqrt{\frac{L_{MAS}}{Q_{ES}'^2 C_{CAS}}} \end{aligned} \quad (18)$$

Or, with more direct approach using electro-mechanical parameters:

$$R_{AE} = \frac{Bl^2}{(R_E + R_g) S_D^2} \quad (19)$$

Similarly, using Q_{MS} we can find value for resistor R_{AM} :

$$R_{AM} = \sqrt{\frac{L_{MAS}}{Q_{MS}^2 C_{CAS}}} = \frac{R_{MS}}{S_D^2} \quad (20)$$

If you added extra mass to your simulation model, then calculated Q_{MS} won't match original driver parameter.

If Q_{ES} and Q_{MS} are not known, then R_{AE} and R_{AM} can be replaced with one resistor R_{AT} representing total Q (Q_{TS}). In that case, calculate its value as follows:

$$R_{AT} = \sqrt{\frac{L_{MAS}}{Q_{TS}^2 C_{CAS}}} \quad (21)$$

Note that using Q_{ES} and Q_{MS} separately leads (at least in principle) to more accurate² value, than using Q_{TS} alone. Following relation is always³ valid for driver Q parameters:

$$\frac{1}{Q_{TS}} = \frac{1}{Q_{ES}} + \frac{1}{Q_{MS}} \quad (22)$$

If you want to calculate impedance response of the driver, then it is mandatory to use separate resistors to determine the acoustic impedance “seen” by the driver, so that electrical impedance may be calculated from the result.

4. Calculate C_{AE} :

This is optional. C_{AE} represents electrical inductance of the voice coil. This kind of simple inductance model doesn’t always work well. For large inductance drivers, the high frequency roll-off effect is too pronounced.

$$C_{AE} = \frac{S_D^2 L_E}{Bl^2} \quad (23)$$

5. Calculate V_{AD} :

V_{AD} represents constant pressure generator, whose value is exactly

$$V_{AD} = \frac{e_G Bl}{(R_E + R_g) S_D} \quad (24)$$

Where e_G is terminal input voltage of the driver, Bl is driver force factor, R_E is the DC-resistance of the voice coil and S_D is driver’s effective radiating surface area. If you are just calculating frequency response, then normalization is performed by relation

$$V_{AD} = |L_{MAS}| \quad (25)$$

If you want to define input as power, then there are more than one possible reference impedance, using what power is calculated. Two common possible values are:

- (a) Use R_E as reference impedance.

This is how Richard Small defined electrical input power, when he derived expression for reference efficiency in his JAES paper[3].

- (b) Use Z_{NOM} as reference impedance.

This approach is used by some box- building programs, (BassBox 6 Pro?⁴)

Whatever your reference impedance level is, value of e_G is

$$e_G = \sqrt{PR} \quad (26)$$

²Difference between two methods is often indistinguishable. Advantage is possibility to calculate electrical impedance.

³Some drivers have ill-conditioned (incorrect) advertised parameters, thus Q- parameters won’t satisfy this equation.

⁴I have recently found good agreement between BassBox 6 pro and WinISD Pro cone excursion calculation results, when power is defined this way.

or power with known drive voltage

$$P = \frac{e_G^2}{R} \quad (27)$$

where R is chosen reference impedance (R_E , Z_{NOM} or whatever you desire).

Note that V_{AD} should be multiplied by 2π only when using microsim⁵, because of need to multiply.

3.3 Full driver model for SPICE-simulations

The model shown in figure 3 can be written as SPICE subcircuit. Often, one needs to model driver in a such way that it can be connected as an electrical component. The complete SPICE-subcircuit for arbitrary driver is as follows:

```
.subckt DRIVER VCP VCN PP PN XDP XDN

* Voice coil circuit

RDCVC VCP 1 { Re }
HBLUD 1 2 VD2 { Bl }
VD1 2 VCN 0

* Mechanical circuit

HBLIC 4 8 VD1 { Bl }
LMMS 4 5 { Mms }
CCMS 5 6 { Cms }
RRMS 6 7 { Rms }
ESDPD 7 PN PP PN { Sd }
VD2 PN 8 0

BVXD XDP XDN V=I(VD2) LAPLACE=1/S

* Acoustical output

FSDUD PN PP VD2 { Sd }

.ends DRIVER
```

VCP and VCN-nodes are for connecting the voice coil. The PP and PN-nodes are output for the enclosure acoustic circuits described in this document. XDP and XDN nodes give out voltage, which is proportional to the cone excursion. If you use basic SI-units(recommended), then 1 volt corresponds to 1 meters excursion, or 1 millivolt corresponds to 1 millimeter excursion. Scaling depends how you set driving voltage source. If input voltage source is RMS volts, then output will be RMS excursion. In that case, multiply excursion voltage by $\sqrt{2}$. Correspondingly, if voltage source is peak, then excursion is peak also. In that case, divide excursion by $\sqrt{2}$ to get the RMS value. Note that calculated acoustic output also changes. By the convention, acoustic output is calculated in dB Pascals RMS referred to 20 μ Pa.

You must provide values for R_E , Bl , M_{MS} , C_{MS} , R_{MS} and S_D . So, your SPICE netlist should contain instantiation line like

⁵This is to ease effort, when using probe. You don't need enter 2π factor into probe expressions then.

```
XU1 In 0 N003 0 N004 0 Driver Mms=135.4m Cms=523.7u Rms=6.11371+  
Re=3.4 Bl=18.9 Sd=352.4e-4
```

There are some possible improvements to the model described above, for example more realistic lossy inductance models. Like one in Leach textbook.

4 Enclosure models

Here I'll present models for some of the boxes. Because of high variability of various bandpass enclosures, only most common types are presented. Current through circuit branches represent a volume velocity through components in question, and unit is m^3/s . Volume velocity is defined by (6). Commonly neglected factor is the radiation impedance. That can be done with quite successfully, because it can be modelled quite easily with just some RLC-components with quite high degree of accuracy. But neglecting these won't usually cause very dramatic effect due to fact that values are quite small compared to other circuit elements anyway. The driver components are not included. Instead, the driver connection points are marked as P_D+ and P_D- .

4.1 Simple rules for drawing equivalent circuits

It is easy to draw equivalent circuits by using the following rules:

1. Ground potential is box surrounding air.
2. Mark each chamber as capacitor to ground and optional resistor in series for absorption losses.
3. Leaks can be modelled with resistor from chamber to "outside".
4. Each port is modelled as inductor and optional resistor in series for losses.
5. Replace driver as circuit shown in figure 4.

Then connect all nodes in the same chamber. Now, when you "straighten" the circuit up, you get your model, which you can enter into your favorite circuit simulator.

4.2 Worked example of equivalent circuit derivation

For example, let's use 8th order bandpass box as example (B**e has marketed such boxes).

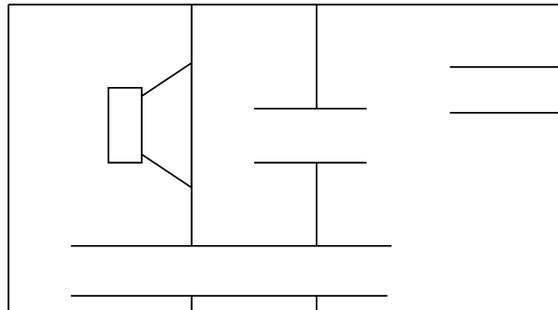


Figure 5: Eighth-order bandpass enclosure schematical drawing.

After drawing the components with above given guidelines, we get schematic overlaid with box structure as shown in figure 6. Only nonideality included are the leak losses modelling resistors⁶. The driver subcircuit is again omitted.

Straightforward, isn't it? And when we "straighten" the circuit up, we get circuit shown in figure 7.

⁶Only simplest model is shown here, further analysis is left as an exercise to the reader.

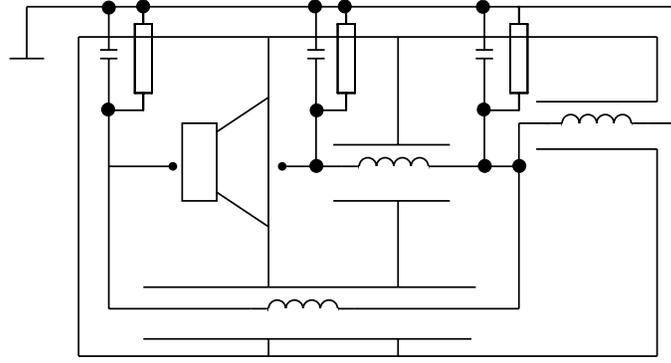


Figure 6: Box schematical drawing and overlaid equivalent circuit.

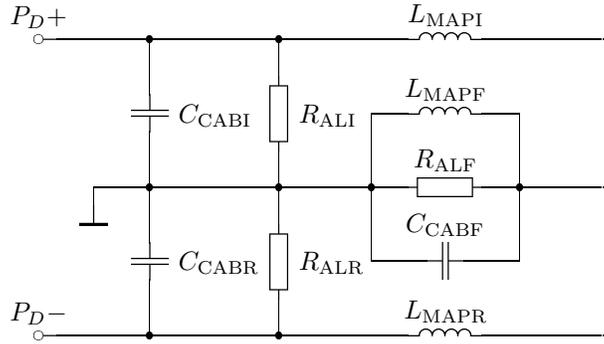


Figure 7: Straightened equivalent circuit of BP8 enclosure.

4.3 Closed box

This is the simplest boxed speaker. It consists only of the box and the driver. Picture of a such box is represented below. Box equivalent circuit with leak and absorption losses is represented in figure 8b.

R_{AB} describes absorptive losses caused by damping material inside speaker cabinet. Typical Q_{AB} for this resistor is about 30-80. R_{AL} describes losses caused by leaks in cabinet. Typically it has value about 5..20 for typical enclosure. Leaks in cabinet are the most dominant loss mode in typical cabinet.

To calculate C_{CAB} value from box volume V_B (actually just as in driver V_{AS} case).

$$C_{CAB} = \frac{V_B}{\rho c^2} \quad (28)$$

Total acoustic capacitance (C_t) in this case is therefore obtained by

$$\frac{1}{C_t} = \frac{1}{C_{CAS}} + \frac{1}{C_{CAB}} \Leftrightarrow C_t = \frac{C_{CAS}C_{CAB}}{C_{CAS} + C_{CAB}} \quad (29)$$

and final resonant frequency without losses (f_{SC}) is

$$f_{SC} = \frac{1}{2\pi\sqrt{C_t L_{MAS}}} \quad (30)$$

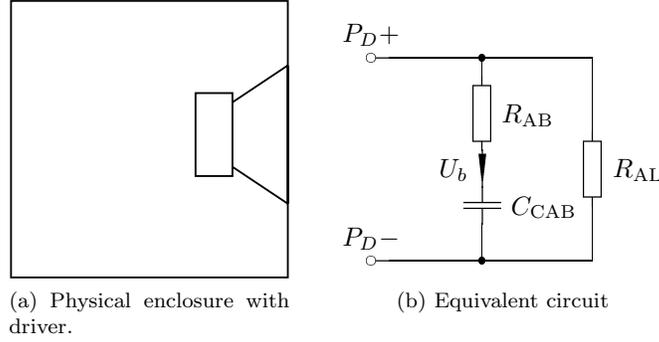


Figure 8: The closed box.

The ratio of f_{SC} to f_s is

$$\begin{aligned}
 \frac{f_{SC}}{f_s} &= \frac{1}{2\pi\sqrt{C_t L_{MAS}}} \frac{2\pi\sqrt{C_{CAS} L_{MAS}}}{1} = \sqrt{\frac{C_{CAS}}{C_t}} = \sqrt{C_{CAS} \frac{C_{CAS} + C_{CAB}}{C_{CAS} C_{CAB}}} \\
 &= \sqrt{\frac{C_{CAS} + C_{CAB}}{C_{CAB}}} = \sqrt{1 + \frac{C_{CAS}}{C_{CAB}}} = \sqrt{1 + \frac{\frac{V_{AS}}{\rho c^2}}{\frac{V_B}{\rho c^2}}} = \sqrt{1 + \frac{V_{AS}}{V_B}} \\
 &\Leftrightarrow f_{SC} = \sqrt{1 + \frac{V_{AS}}{V_B}} \quad (31)
 \end{aligned}$$

By making the following substitutions

$$R_t = R_{AE} + R_{AM} \quad (32)$$

and

$$\omega_{sc} = 2\pi f_{SC} \quad (33)$$

Total Q_{TC} of the final box is (without any absorption and leak losses)

$$Q_{TC} = \frac{1}{\omega_{sc} C_t R_t} = \frac{\sqrt{L_{MAS} C_t}}{C_t R_t} = \sqrt{\frac{L_{MAS}}{C_t R_t^2}} \quad (34)$$

Leak resistor is a bit different, because it only considers C_{CAB} , not C_t as R_{AB} does.

$$Q_l = \omega_{sc} C_{CAB} R_{AL} \Leftrightarrow R_{AL} = \frac{Q_l}{\omega_{sc} C_{CAB}} \quad (35)$$

So designing a closed box is basically tuning this series LCR-circuit to desired Q (Q_{TC}). During that process, total resonant frequency always increases, because acoustic capacitances of the driver and enclosure connect in series, so resultant capacitance is always smaller than either one of the capacitances by itself. With this background, it is easy to see why the drivers which are meant to be installed as “free-air” have rather large Q_{TS} : in case of free-air installation, capacitance C_{CAB} becomes “infinitely” large (“infinite” volume). So it by itself has desired Q of about 0.6-0.9. Please see following chapter about infinite baffle about this.

The closed box is a third order model if all the losses are included. It is thus not possible to derive a simple Q_{TC} value formula for model with leak losses. This is because resulting transfer function will have three poles instead of two. Definition of Q defines Q for pole pair. Thus, it is not possible.

4.4 Infinite Baffle

Infinite baffle is in fact a special case of the sealed box, because it can be formed from sealed box by enlarging the enclosure out to infinity. Of course, you can't build such a big baffle, so practical implementations are more or less an approximation to true infinite baffle.

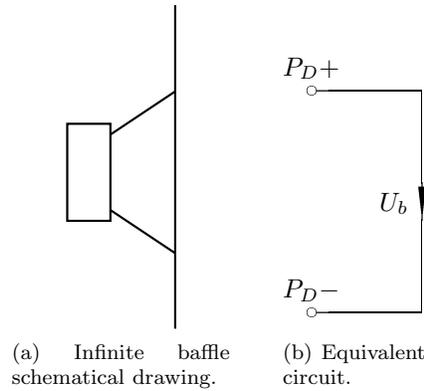


Figure 9: The infinite baffle.

Of course, the short circuit is replaced by the radiation impedance.

4.5 Ported box (bass reflex)

The ported box basically is a Helmholtz resonator, which is excited by the driver. It enhances the low-frequency output with this resonance. Schematical representation of ported box is shown in figure 10a. Unfortunately, it causes some undesirable side-effects, poorer transient response being one compared to closed box. This is mainly due to steeper roll-off than in closed box. Because phase response is so called Hilbert transform of amplitude response for any minimum-phase system⁷, amplitude response determines system transient response. The transient response can be improved by suitable design of the response. This kind of alignment is so called SBB4 alignment family. Other drawbacks are possible spurious noises generated by the flowing air at high output levels.

Rational design of ported boxes have not been existing for very long time. Thiele[5] was first to introduce so called "alignments" to general public. Design of the ported box involves two parameters, compliance ratio α and tuning ratio h .

They are defined by relations

$$h = \frac{f_B}{f_s} \Leftrightarrow f_B = h f_s \quad (36)$$

and

$$\alpha = \frac{V_{AS}}{V_B} \Leftrightarrow V_B = \frac{V_{AS}}{\alpha} \quad (37)$$

The tuning frequency can be derived as is the case for electrical parallel resonant circuit. In resonance, reactance of capacitance is equal to reactance of inductance. Because parallel resonance circuit looks like open at resonance, driver sees very high acoustic load. Thus its excursion will reduce considerably around box tuning frequency f_B . Frequencies below f_B , acoustic load will diminish, and the cone excursion will grow, thus power handling will reduce considerably below f_B .

⁷Minimum-phase system refers to system which has all its zeros in left-half of s-plane.

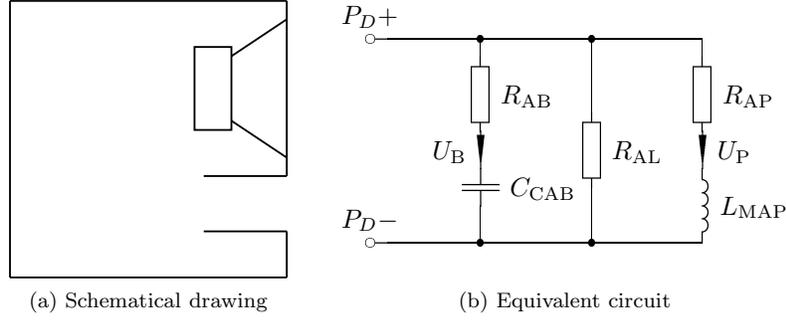


Figure 10: Ported box.

Equivalent circuit in figure 10 is complete with absorption and port losses in addition to leak loss described by Small. These losses are usually determined with aid of various Q-values.

$$\bar{Z}_C = \bar{Z}_L \Leftrightarrow \frac{1}{sC} = sL \quad (38)$$

Solving for s ($j\omega$) we get

$$s = \frac{1}{\sqrt{LC}} \quad (39)$$

thus, resonance frequency is

$$f_b = \frac{1}{2\pi\sqrt{LC}} \quad (40)$$

Acoustic capacitance of the box is defined by (28). Acoustic impedance of the open-ended tube is

$$\bar{Z}_A = j\frac{\rho c}{S_v} \tan\left(\frac{\omega l}{c}\right) \quad (41)$$

Now, we can use small argument approximation for tan-function. When tangent function argument is small, we have $\tan(x) \simeq x$. Therefore for acoustic inductance of the port, general low-frequency approximation is

$$\bar{Z}_A = j\omega \frac{\rho l_t}{S_v} = s \frac{\rho l_t}{S_v} \quad (42)$$

And the acoustic inductance can be defined as

$$L_{MAP} = \frac{\rho l_t}{S_v} \quad (43)$$

where l_t is acoustical length of the port, i.e. it includes end correction parts, S_v is effective radiating area of the port, i.e. cross-section area, and L_{MAP} is the acoustic inductance.

By substituting (28) and (43) into (40) we get

$$f_B = \frac{1}{2\pi\sqrt{\frac{\rho l_t}{S_V} \cdot \frac{V_B}{\rho c^2}}} = \frac{1}{2\pi\sqrt{\frac{l_t V_B}{S_V c^2}}} = \frac{c}{2\pi\sqrt{l_t V_B}}$$

$$\Leftrightarrow l_t = \frac{c^2}{(2\pi)^2} \cdot \frac{A}{f_B^2 V_B} \quad (44)$$

To generalize end correction to any shape, it is commonplace to replace the port radius with expression

$$r_V = \sqrt{\frac{S_V}{\pi}}. \quad (45)$$

End correction is added to length, so that physical length of the port becomes

$$l_V = l_t - kr_V \Leftrightarrow l_t = l_V + (k_1 + k_2)r_V \quad (46)$$

Then, we can obtain following expression for tuning frequency:

$$f_B = \frac{c}{2\pi\sqrt{\frac{S_V n_p}{\left(l_V + k\sqrt{\frac{S_V}{\pi}}\right) V_B}}} \Leftrightarrow f_B = \sqrt{\frac{S_V n_p}{V_B \left(l_V + (k_1 + k_2)\sqrt{\frac{S_V}{\pi}}\right)}} \quad (47)$$

where k_1 and k_2 are end corrections of each end, described as terms of effective port radius.

To determine component values for ported box, use same methodology than for closed box. Tuning frequency and acoustic capacitance C_{CABR} determine value of port modelling inductor L_{MAP} . Thus, this inductor's value is defined to be

$$f_B = \frac{1}{2\pi\sqrt{L_{MAP} C_{CABR}}} \Leftrightarrow L_{MAP} = \frac{1}{(2\pi f_B)^2 C_{CAB}} \quad (48)$$

To determine loss-modelling resistors R_{AB} (absorption) and R_{AP} (port friction), one needs Q-values, Q_A and Q_P . Typical Q_P for port is about 80..100, and Q_A for unlined enclosure is about 100.

Equation (11) can be used to determine R_{ALR} , the leak modelling resistor. Series resonance circuit Q was defined in equation (10). For convenience, R_{ABR} and R_{APR} can be calculated using equation

$$R = \frac{1}{Q\omega_B C} = \frac{1}{2\pi f_B Q C} \quad (49)$$

where Q is either Q_{ABR} or Q_{APR} and f_B is box tuning frequency.

4.6 Worked example of the ported box

As an example, the component values for specific ported box is determined. The box has the following parameters, shown in table 1.

First, C_{CAB} is calculated:

$$C_{CAB} = \frac{V_B}{\rho c^2} = \frac{100 \times 10^{-3}}{1.20 \times 343.68^2} \approx \underline{7.0552 \times 10^{-7}}$$

Table 1: Ported box parameters

Parameter	Value	Unit
V_B	100	1
f_B	25	Hz
Q_A	20	
Q_L	10	
Q_P	100	

Then, L_{MAP} is determined from f_B and C_{CAB} :

$$L_{MAP} = \frac{1}{(2\pi f_B)^2 C_{CAB}} = \frac{1}{(2\pi 25)^2 7.0552 \times 10^{-7}} \approx \underline{57.4447}$$

Now we can determine the loss modelling resistor values, R_{AL} , R_{AB} and R_{AP} :

$$R_{AL} = \frac{Q_L}{2\pi f_B C} = \frac{10}{2\pi 25 \times 7.0552 \times 10^{-7}} \approx \underline{90234.1}$$

$$R_{AB} = \frac{1}{2\pi f_B Q_{AB} C} = \frac{1}{2\pi 25 \times 20 \times 7.0552 \times 10^{-7}} \approx \underline{451.17}$$

$$R_{AP} = \frac{1}{2\pi f_B Q_P C} = \frac{1}{2\pi 25 \times 100 \times 7.0552 \times 10^{-7}} \approx \underline{90.234}$$

Figure 11 shows the simulated magnitude of the frequency response of the example box with Tymphany XLS-12 driver. The plot also shows the contribution of the port to the response and cone excursion. Input level to the driver terminals is 30 volts RMS. It can be seen that all the sound is radiated from the port near the tuning frequency.

As the graph shows, the cone excursion X_D skyrockets rapidly below f_B . Therefore it is desirable to have low tuning frequency. However, if the tuning frequency is too low, the peak excursion above f_B will get too high.

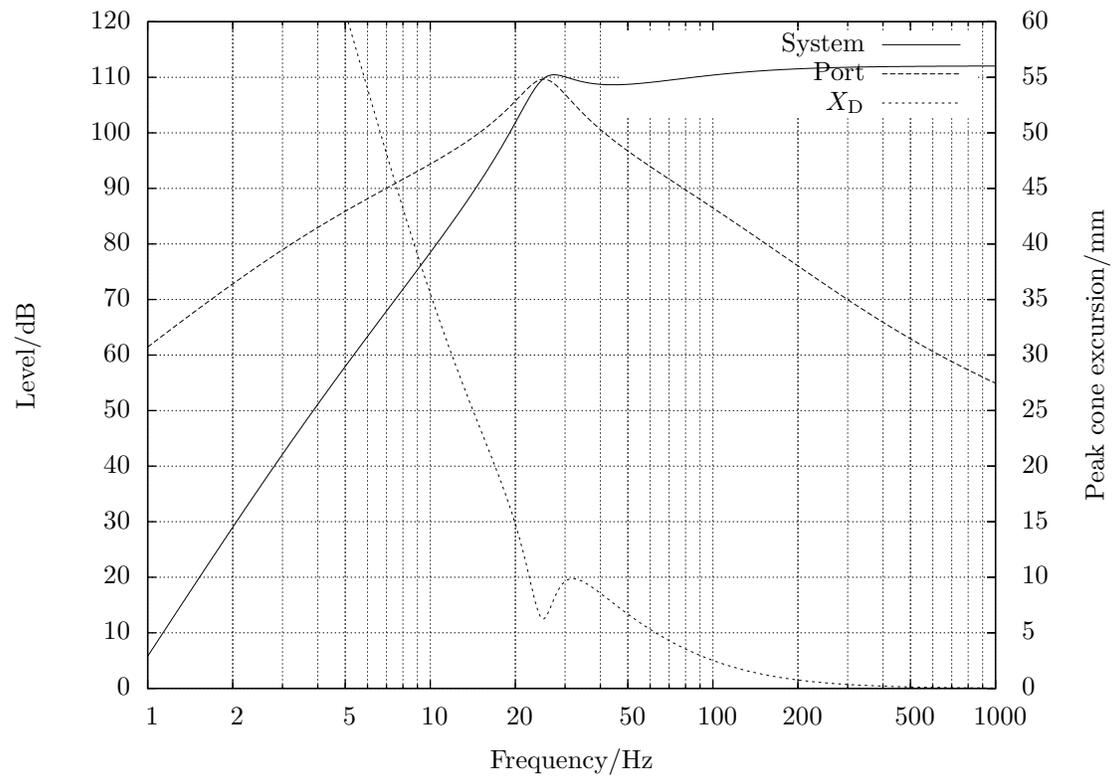


Figure 11: Simulated frequency response for the example box with Tymphany XLS-12 driver.

4.7 Passive radiator box

Passive radiator box is a variation of ported box. Instead of the air in tube, the mass is included in moving diaphragm, like in normal plain driver. This prevents some problems of the port, like chuffing sound at high port velocities. Another, and may be the most important advantage is that very low tuning frequencies can be realized successfully, whereas conventional port becomes awkwardly long. Drawback of the passive radiator is that extra compliance C_{AR} adds extra zero at the frequency response, creating a null at resonance frequency of the passive radiator resonance frequency. With proper tuning, the effect can be optimized to have little or no harmful effect. It should be remembered that passive radiator radiates all the sound at Helmholtz frequency, so there should be enough volume displacement.

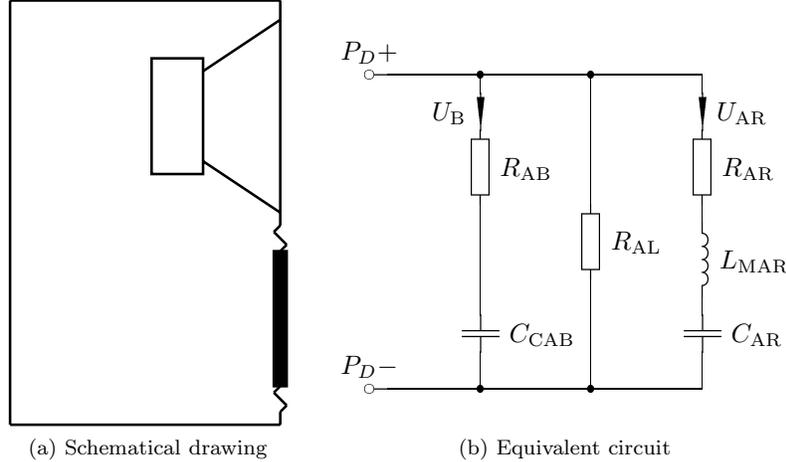


Figure 12: Passive radiator box.

Tuning (Helmholtz resonance) frequency for passive radiator box can be obtained as follows:

$$V'_B = \frac{1}{\frac{1}{V_{AP}} + \frac{1}{V_B}} = \frac{V_{AP}V_B}{V_{AP} + V_B} \quad (50)$$

Then, by just using (40), we get

$$\begin{aligned} f_b &= \frac{1}{2\pi \sqrt{\frac{M_{MP}V'_B}{S_P^2 \rho c^2}}} = \frac{c\sqrt{\rho}}{2\pi} \frac{S_P}{\sqrt{M_{MP}V'_B}} \\ &\Leftrightarrow M_{MP} = \frac{c^2 \rho}{(2\pi)^2} \frac{S_P^2}{f_b^2 V'_B} \end{aligned} \quad (51)$$

Here M_{MP} and M_{MS} are used interchangeably. Passive radiator is just like active driver, but V_{AD} and R_{AE} is omitted. Thus, circuit elements relating to electrical circuit are omitted. On other words, add capacitance corresponding to the passive radiator equivalent volume V_{AS} in series of conventional port, and change values of the respective circuit elements correspondingly.

The passive radiator parameters can be determined in various ways. The basic procedure is just like with the normal active driver. The most direct way is to use mechanical parameters C_{MS} , R_{MS} and M_{MS} . Alternatively, V_{AS} and environmental parameters can be used to determine the parameters.

$$C_{AR} = C_{MS} S_D^2 = \frac{V_{AS}}{\rho c^2} \quad (52)$$

$$L_{\text{MAR}} = \frac{M_{\text{MS}}}{S_{\text{D}}^2} = \frac{1}{(2\pi f_{\text{s}})^2 C_{\text{AR}}} = \frac{\rho c^2}{(2\pi f_{\text{s}})^2 V_{\text{AS}}} \quad (53)$$

$$R_{\text{AR}} = \frac{1}{f_{\text{s}} Q_{\text{MS}} C_{\text{AR}}} = \frac{\rho c^2}{f_{\text{s}} Q_{\text{MS}} V_{\text{AS}}} \quad (54)$$

As an example, let's determine the parameters for Tymphany 830548, which is a 12" passive radiator, which is primarily used with Tymphany XLS-12 subwoofer driver, type 830500. Parameters are in table 2. As throughout this paper, standard values of c (343.68 m/s) and ρ (1.20 kg/m³) are used.

Table 2: Tymphany 830548 Parameters.

Parameter	Value	Unit
f_{s}	10.4	Hz
Q_{MS}	15.2	
V_{AS}	169.6	l
M_{MS}	425	g
C_{MS}	0.55	mm/N
S_{D}	466	cm ²

To show the effect on the parameters, calculations will be done using all forms of the equations (52-54), where applicable.

1. Calculation of C_{AR}

(a) Using S_{D} and C_{MS} :

$$C_{\text{AR}} = C_{\text{MS}} S_{\text{D}}^2 = 0.55 \times 10^{-3} \times (466 \times 10^{-4})^2 \approx \underline{1.19436 \times 10^{-6}}$$

(b) Using V_{AS} , ρ and c

$$C_{\text{AR}} = \frac{V_{\text{AS}}}{\rho c^2} = \frac{169.6 \times 10^{-3}}{1.20 \times (343.68)^2} \approx \underline{1.19656 \times 10^{-6}}$$

2. Calculation of L_{MAR}

(a) Using M_{MS} and S_{D}

$$L_{\text{MAR}} = \frac{M_{\text{MS}}}{S_{\text{D}}^2} = \frac{425 \times 10^{-3}}{(466 \times 10^{-4})^2} \approx \underline{195.712}$$

(b) Using ρ , c , f_{s} and V_{AS}

$$L_{\text{MAR}} = \frac{\rho c^2}{(2\pi f_{\text{s}})^2 V_{\text{AS}}} = \frac{1.20 \times 343.68^2}{(2\pi 10.4)^2 169.6 \times 10^{-3}} \approx \underline{195.721}$$

3. Calculation of R_{AR}

(a) Using f_s , Q_{MS} and C_{AR}

$$R_{AR} = \frac{1}{2\pi f_s Q_{MS} C_{AR}} = \frac{1}{2\pi 10.4 \times 15.2 \times 1.19436 \times 10^{-6}} \approx \underline{842.96}$$

(b) Using ρ , c , Q_{MS} and V_{AS}

$$R_{AR} = \frac{\rho c^2}{f_s Q_{MS} V_{AS}} = \frac{1.20 \times 343.68^2}{2\pi 10.4 \times 15.2 \times 169.6 \times 10^{-3}} \approx \underline{841.41}$$

Results are pretty much equal for any practical purpose, considering the manufacturing tolerances involved. However, it should be clear that although it is irrelevant which parameters are being used when determining the model parameters, they are not exactly the same, due to numerical round off errors. And of course, unless values of c and ρ are not given, there is no way to tell what the air properties were when the value of V_{AS} was calculated from C_{MS} .

To illustrate the response characteristics, the response is simulated in 20 liters box with the typical driver with that the this particular PR might be used with. The effective volume in the resonator is

$$V'_B = \frac{V_{AP} V_B}{V_{AP} + V_B} = \frac{169.6 \times 20}{169.6 + 20} \approx 17.891$$

$$C'_{CAB} = \frac{V'_B}{\rho c^2} = \frac{17.89 \times 10^{-3}}{1.20 \times 343.68^2} \approx 1.2622 \times 10^{-7}$$

The Helmholtz resonance (tuning) frequency then becomes by using (51)

$$f_B = \frac{c\sqrt{\rho}}{2\pi} \frac{S_P}{\sqrt{M_{MS} V'_B}} = \frac{343.68\sqrt{1.20}}{2\pi} \frac{466 \times 10^{-4}}{\sqrt{425 \times 10^{-3} 17.89 \times 10^{-3}}} \approx 32.0 \text{ Hz}$$

Assuming absorption quality factor of 100 and leak quality factor of 7, the loss resistor values are computed as

$$R_{AL} = \frac{Q_L}{2\pi f_B C'_{CAB}} = \frac{7}{2\pi 32 \times 1.2622 \times 10^{-7}} \approx \underline{275833.9}$$

$$R_{AB} = \frac{1}{2\pi f_B Q_{AB} C'_{CAB}} = \frac{1}{2\pi 32 \times 100 \times 1.2622 \times 10^{-7}} \approx \underline{394.04}$$

Figure 13 shows the simulated system response and cone excursion for the active driver. The driver is again Tymphany XLS-12, and box size is 20 litres. The characteristic null at the passive radiator resonance frequency is also clearly shown. Otherwise the response characteristics is quite similar that of the ordinary ported box. The null in the driver excursion around Helmholtz frequency is not so sharp as it is with ported box. This, however depends upon the passive radiator and how good the construction of the ported box is. Generally speaking, the quality factor of the passive radiator is in order of the decade lower than the port.

To be continued...

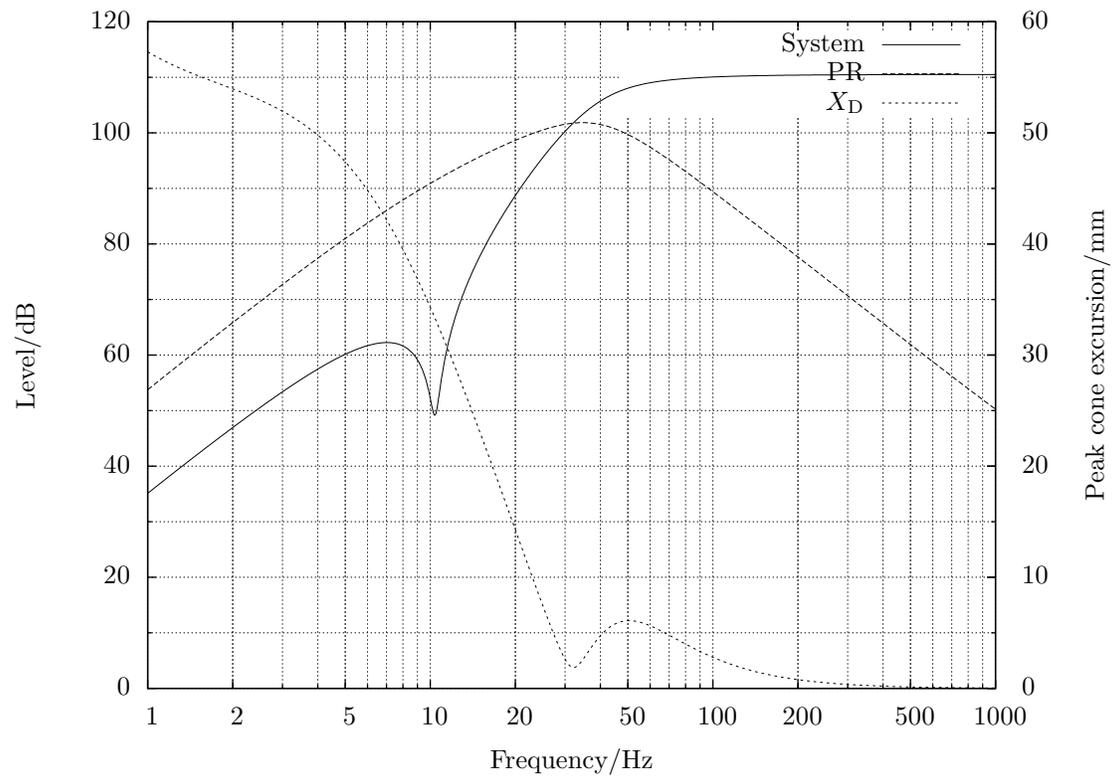


Figure 13: Simulation for the 20 l PR box with Tymphany 830500 driver.

4.8 Fourth-order bandpass box

Fourth order bandpass box is shown in the figure 14 along with the equivalent circuit. It is the simplest bandpass box. It basically consists of the Helmholtz resonator put in front of conventional closed box. All output will be from the port of the Helmholtz resonator, so very high outputs can be achieved if the bandwidth of the system is sufficiently narrow. This is in fact characteristic feature of all bandpass boxes; the bandwidth and gain can be exchanged for each other.

The box attenuates high frequencies by design. This might lead to conclusion that the lowpass filter would not be necessary with the bandpass box. The lowpass will work only upto the frequency where front port behaves as lumped mass. This frequency is about the frequency where wavelength is four times the port physical length. The first port resonance frequency is just at that frequency. Unless higher frequencies are filtered, they will be output from the port.

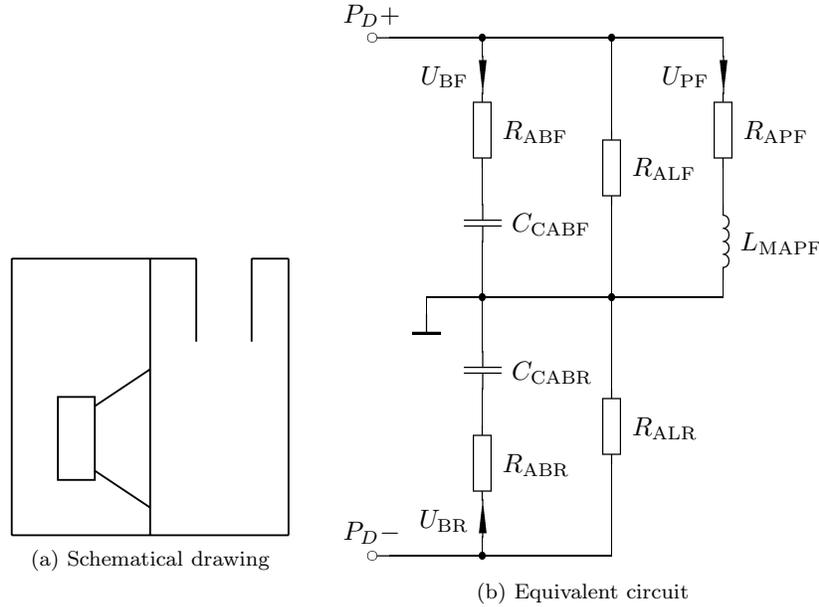


Figure 14: 4th order bandpass box.

Table 3: 4th Order bandpass example box parameters

Parameter	Value	Unit
V_{BF}	40	1
f_{BF}	30	Hz
Q_{AF}	100	
Q_{LF}	10	
Q_{PF}	100	
V_{BR}	70	1
Q_{LR}	10	
Q_{AR}	100	

The bandpass box with Tympany 830500 is simulated as an example. The model described here doesn't include interchamber leak modelling resistor, which could be included, if necessary. The box is then re-simulated with front port modelled as transmission line terminated to radiation impedance to show the effect of the port resonances. From the simulation it is clear that the electrical low-pass filter is in practice mandatory, if clean bass reproduction is desired.

The method of calculating equivalent circuit component values for the front chamber is exactly same as in the ported box case, described in the section 4.6. Again, first the C_{CABF} ⁸ is calculated:

$$C_{CABF} = \frac{V_{BF}}{\rho c^2} = \frac{40 \times 10^{-3}}{1.20 \times 343.68^2} \approx \underline{2.8220 \times 10^{-7}}$$

Then, L_{MAPF} is determined from f_{BF} and C_{CABF} :

$$L_{MAPF} = \frac{1}{(2\pi f_{BF})^2 C_{CABF}} = \frac{1}{(2\pi 30)^2 2.8220 \times 10^{-7}} \approx \underline{99.730}$$

Then, the front chamber loss resistors can be calculated:

$$R_{ALF} = \frac{Q_L}{2\pi f_{BF} C} = \frac{10}{2\pi 30 \times 2.8220 \times 10^{-7}} \approx \underline{187993.1}$$

$$R_{ABF} = \frac{1}{2\pi f_{BF} Q_{AF} C} = \frac{1}{2\pi 30 \times 100 \times 2.8220 \times 10^{-7}} \approx \underline{187.99}$$

$$R_{APF} = \frac{1}{2\pi f_{BF} Q_{PF} C} = \frac{1}{2\pi 30 \times 100 \times 2.8220 \times 10^{-7}} \approx \underline{187.99}$$

For calculation of the rear chamber loss components, the f_{SC} of the rear chamber with the driver is calculated.

$$V'_{BR} = \frac{V_{BR} V_{AS}}{V_{BR} + V_{AS}} = \frac{70 \times 139.2}{70 + 139.2} \approx \underline{46.58} \Rightarrow C'_{CABR} \approx 3.2861 \times 10^{-7}$$

Actual C_{CABR} is

$$C_{CABR} = \frac{V_{BR}}{\rho c^2} = \frac{70 \times 10^{-3}}{1.20 \times 343.68^2} \approx \underline{4.9387 \times 10^{-7}}$$

And the resultant resonance frequency for rear chamber and the driver is by using (31)

$$f_{SC} = f_s \sqrt{1 + \frac{V_{AS}}{V_B}} = 18.1 \text{ Hz} \sqrt{1 + \frac{139.2}{70}} \approx 31.29 \text{ Hz}$$

And the rear chamber leak and absorption loss modelling resistors:

$$R_{ALF} = \frac{Q_L}{2\pi f_{SC} C} = \frac{10}{2\pi 31.29 \times 3.2861 \times 10^{-7}} \approx \underline{154786.8}$$

$$R_{ABF} = \frac{1}{2\pi f_{SC} Q_{AF} C} = \frac{1}{2\pi 31.29 \times 100 \times 3.2861 \times 10^{-7}} \approx \underline{154.787}$$

Now, for the simulation of the front port as an transmission line, we must first determine the port diameter and thus its length. For port diameter of 102 mm, the approximate port length is about 310 mm.

⁸Subscript F means front chamber, and R means rear chamber.

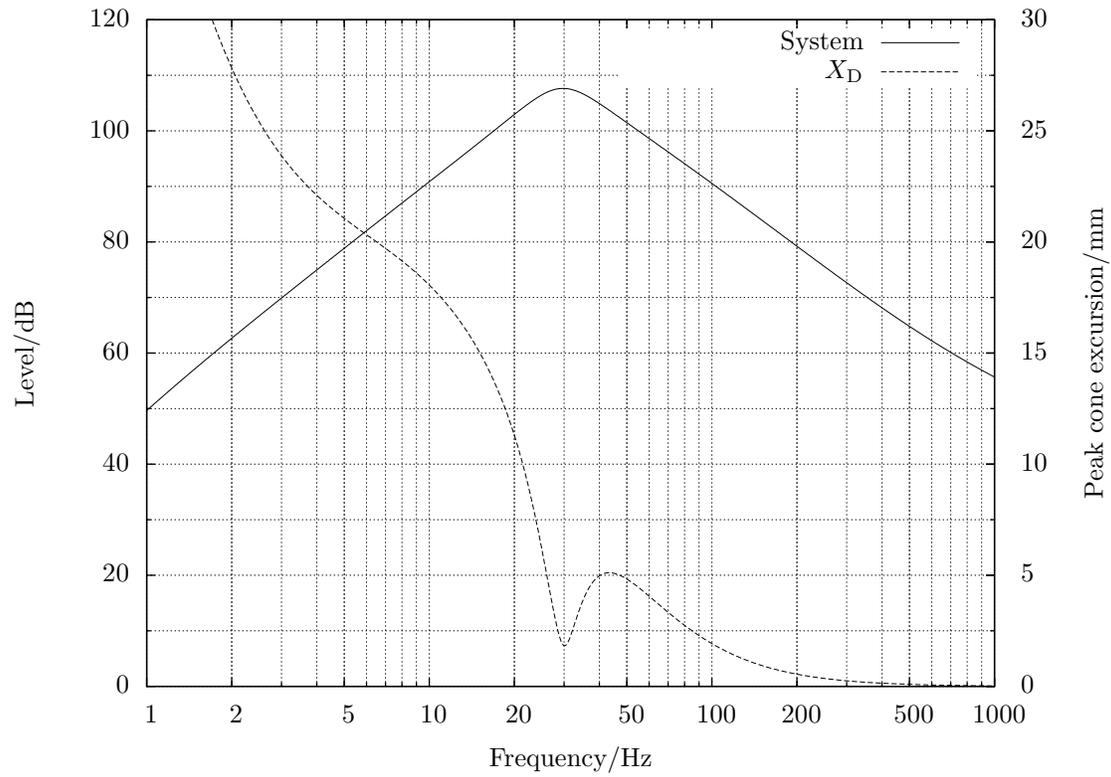


Figure 15: Simulation for the 30+70 l box with Tymphany 830500 driver.

4.9 fourth-order bandpass box with passive radiator

It is possible to replace the front port from the fourth order bandpass enclosure with passive radiator. This eliminates the port resonances, but the cavity resonances of the front chamber will still remain. 4th order bandpass box with passive radiator is shown in figure 16 along with corresponding equivalent circuit.

Of course, the concept of replacing the port with passive radiator is universal concept, that can be applied to any box topology which uses ports. The effect of passive radiator self resonance must be considered separately in each case. And, it is sometimes difficult to obtain same tuning with passive radiator than with port, due to that passive radiator moving mass is easily added, but not removed. To be continued...

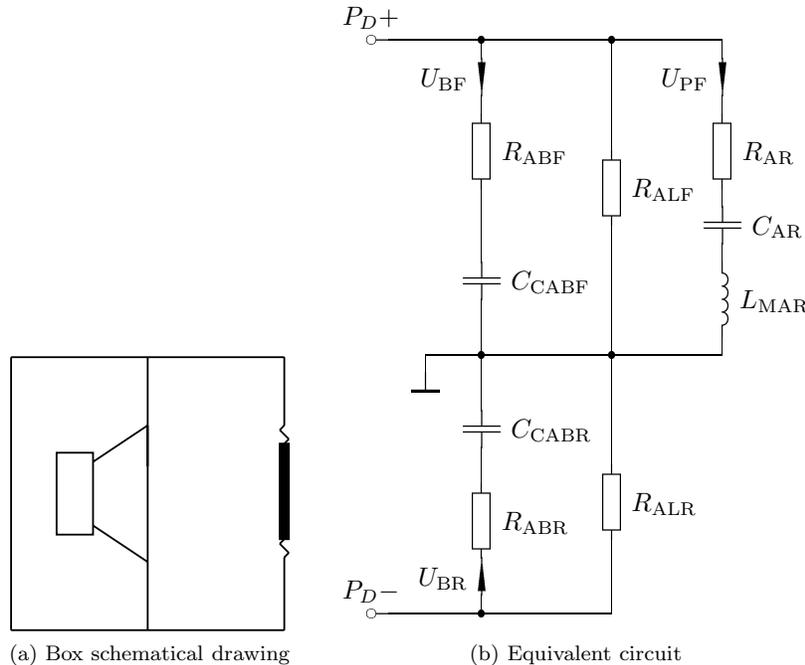


Figure 16: 4th order bandpass box with passive radiator.

4.10 Sixth-order bandpass box (two external ports)

To make things more advanced, we can port both chambers, and that way we get sixth-order bandpass. Equivalent circuit is two parallel resonant circuits in series of the driver circuit. Each parallel resonant circuit represents a chamber. Selection which one is front and which one is rear, is quite arbitrary, since driver has no correct polarity at low frequencies. This kind of box is shown in figure 17.

The design requires that rear and front chambers must not be exactly the same. Otherwise there is no output from the box. This is easy to understand as the chambers would then produce identical response, but out of phase, thus sum is zero. To be continued...

4.11 Sixth-order bandpass box (one internal and one external port)

Sixth order bandpass box can be also configured so that the other port is internal to the box. The equivalent circuit and box drawing is in figure 18. To be continued...

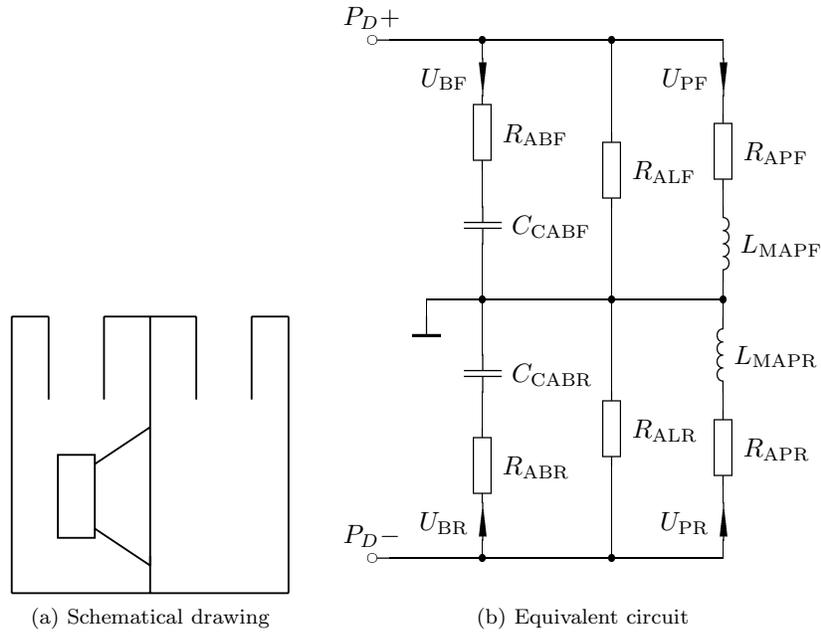


Figure 17: Sixth-order bandpass box.

4.12 Two-chamber bandpass with two drivers

To be continued...

4.13 The ABC Box

The ABC Box⁹ shown in figure 19 is just like conventional bass reflex box, but the rear chamber is split into two different chambers, and connected with port. Also, the chamber without driver is also made ported. For frequency response, this makes a notch in response followed by considerable peaking. This property makes this type of box suitable in SPL applications where acceptable reproduction of other frequencies is also required.

Simulation of this type of box is not so straightforward if you use conventional simulation programs like BassBox. The only software I know that is directly capable of simulating this type of box, is WinISD pro 0.60 upwards.

⁹ABC is acronym for Aperiodic Bi-Chamber, which is technically incorrect, there is no resistive damping, as-is.

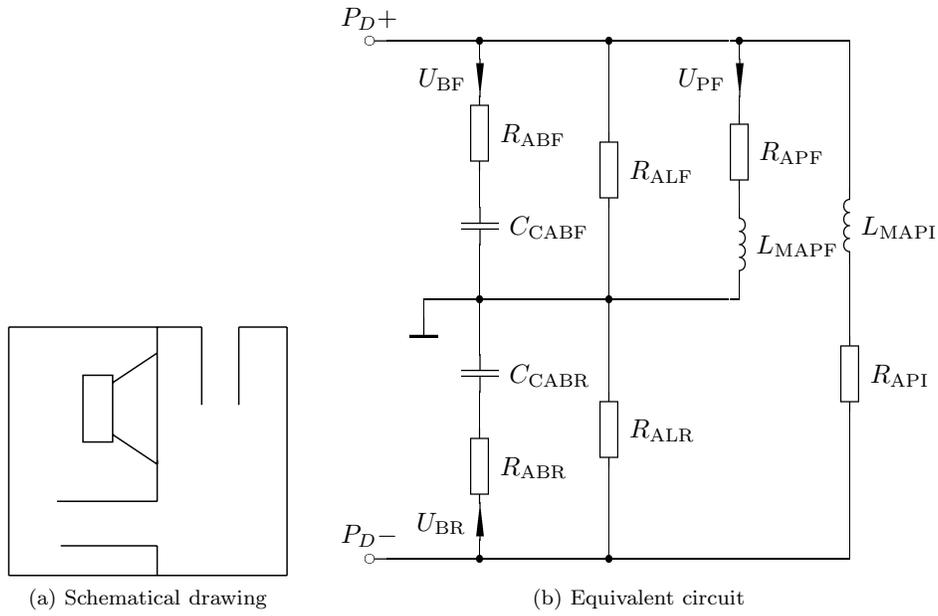


Figure 18: 6th order bandpass box with one internal port.

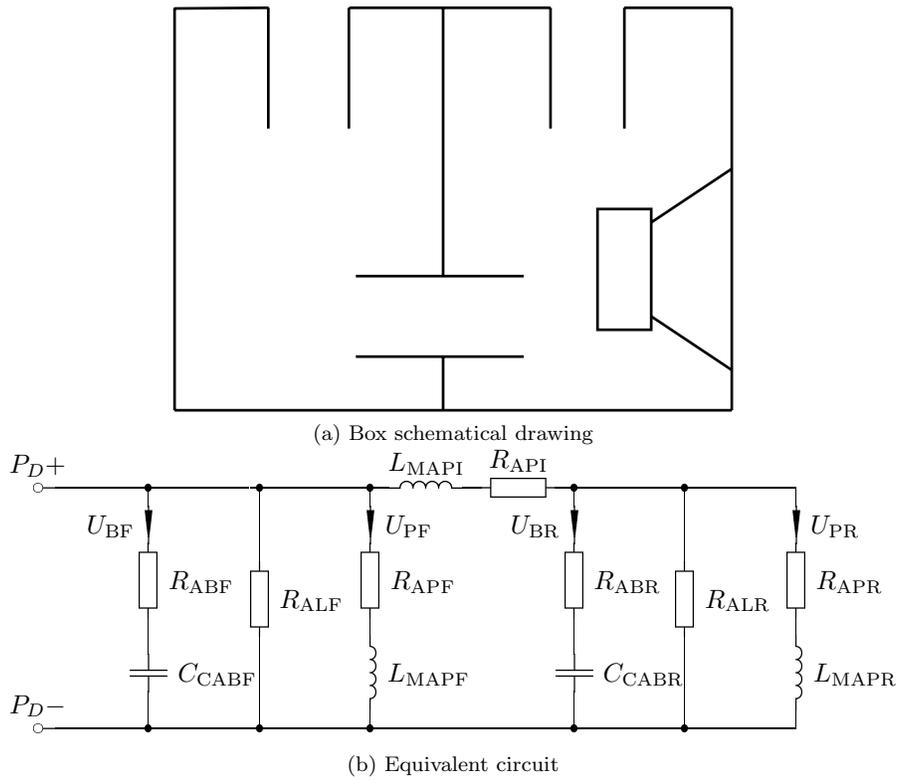


Figure 19: The ABC Box.

5 Simulating circuit models with PSPICE

You can simulate calculated models with some circuit simulator. I'll include examples with microsim (version 8) here. Once you get grasp of this concept, you can quite easily simulate your craziest box idea. Basically, following steps are required to do this:

1. Draw equivalent circuit into microsim
2. Calculate circuit component values
Note that above calculated component values can be entered directly into microsim, any kind of conversion is not necessary.
3. Set microsim to AC-sweep simulation of frequency range of interest
I recommend to use decade or octave sweep, because it produces most rational graphs anyway.
4. Use probe to plot desired graphs

5.1 Frequency response

Generally, total volume velocity, which determines output from any direct radiator enclosure, is the vector sum of all volume velocities of different chambers. This applies to any combination of ports and chambers and passive radiators. Sound pressure level from distance r from monopole source into full space 4π assuming far field conditions can be calculated with. It is required that $r \geq 8S_d/(\pi\lambda)$ to have valid far-field region.

$$p = \frac{\rho s U_0}{4\pi r} e^{jkr} \quad (55)$$

Where U_0 is total volume velocity produced by system, r is distance from monopole source and k is so called wave number, ω/c . ρ is the air density. Exponential part represents only linear phase shift produced by time delay. It is commonly neglected, since it is not very interesting generally. Normally, absolute sound pressure is referenced to 0 dB reference, $20\mu\text{Pa}$.

So, with closed box assuming one meter distance, frequency response or absolute SPL level can be plotted with PSPICE probe expression

```
20*log10(I(Ccab)*2*pi*frequency)
```

Likewise, if there are two chambers, like in 6th and 4th order bandpass enclosures, above expression is

```
20*log10((I(Ccabr)+I(Ccabf))*2*pi*frequency)
```

It is also possible to plot different contributions to the total sound pressure. For example, the SPL produced by the port can be plotted with command:

```
20*log10(I(Lmapr)*2*pi*frequency)
```

5.2 Group delay

Group delay is the negative derivative of phase versus frequency. So it tells how fast phase is changing.

$$g_d(\omega) = -\frac{d\varphi(\omega)}{d\omega} \quad (56)$$

The group delay can be calculated with probe using GD()-function. With the closed box, group delay can be plotted with a probe expression

```
GD(I(Ccab)*2*pi*frequency)
```

5.3 Cone excursion

Cone excursion plots how much driver cone will move, when whole design is fed with specified drive voltage defined by (24), or if one uses more advanced model of the driver described in 3.3, the voltage is just like the amplifier would output it in real world. The cone excursion is integral of volume velocity produced by the driver. Usually, one is interested of peak excursion instead of RMS value.

$$\hat{X}_d(\omega) = \frac{\sqrt{2}}{S_d} \int U_d(\omega) d\omega \quad (57)$$

This expression can be plotted in probe with command

```
I(Lmap)*sqrt(2)/(2*pi*frequency*Sd)
```

where `Sd` must be replaced with actual value of S_d of the driver in question, to obtain correct scaling. Result is also scaled with $\sqrt{2}$ to obtain peak excursion. This is not done (incorrectly) by some programs I have seen ¹⁰. If all units are base SI-units, then cone excursion result will be, as would be expected, in meters.

5.4 Air velocity in port

It follows from definition of volume velocity (6) that we can get air velocity from port volume velocity by just simply dividing the volume velocity by effective port radiating area, S_v .

$$\hat{v} = \sqrt{2} \frac{U_p}{S_v} \quad (58)$$

Where U_p is volume velocity produced by port. Again, peak velocity is obtained by scaling RMS velocity by $\sqrt{2}$. Same method can be used to obtain driver cone velocity, if required.

The probe expression

```
I(Lmap)*sqrt(2)/Sv
```

can be used to plot port velocity where `Sv` must be replaced with effective vent area, i.e. inside area of the port.

5.5 Electrical impedance

Electrical impedance is obtained by calculating acoustical impedance seen by the mechanical system, and then using scaling factor of

$$\bar{Z}_{ea} = \bar{Z}_a \frac{(Bl)^2}{S_d^2} \quad (59)$$

and then calculating final electrical impedance in electrical side as

$$\bar{Z}_e = \bar{Z}_{ea} + sL_e + R_e, \quad (60)$$

where L_e is voice coil inductance and R_e is voice coil DC-resistance.

If using model described in 3.3, then determining the impedance becomes considerably simpler. Just plot the ratio of the driver voltage to the driver terminal current.

¹⁰True Audio WinSpeakerz and BassBox 6 pro fall into this category. Correct me if I am mistaken.

5.6 Deriving transfer functions from equivalent circuits

Although easiest is to use conventional circuit analysis techniques, one sometimes wants to obtain mathematically exact transfer function for the box- driver combination.

To be continued...

References

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- [4] W. M. Leach, "Introduction to electroacoustics and audio amplifier design", ISBN 0-7872-7861-0
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